2. Growth of Functions:

Asymptotic Notation:-

"Asymptotic analysis of an algorithms refers to defining the mathematical boundation on its runtime performance. Using asymptotic analysis we can very well conclude the best case, Average Case and worst Case."

Notations:-

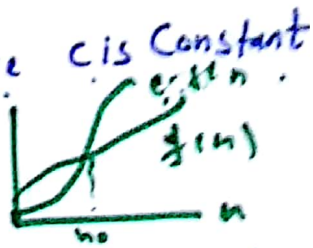
a) Big-oh. Worst Case. Upper Bound (UB)

b) Omega-(Ω). best Case. Lower Bound (LB)

c) Theta. (Θ) - Avg Case - Tight bound (TB)

Big-oh-(O):- if $f(n) \leq c \cdot g(n)$ where c is Constant

$$f(n) = O(g(n))$$



Omega(Ω):- if $f(n) \geq c \cdot g(n)$ where c is Constant

$$f(n) = \Omega(g(n))$$

Theta (Θ) - if $c_1 g(n) \leq f(n) \leq (c_2 \cdot g(n))$

$$f(n) = \Theta(g(n))$$

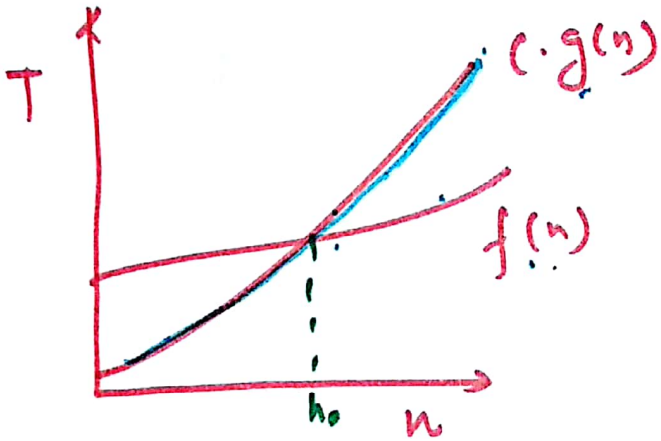
(Asymptotic Notations)

Big (O):- oh - Worst Case - Upper Bound (UB)

Omega (Ω) - Best Case. Lower Bound (LB)

Theta (Θ) - Avg Case: Tight Bound (TB)

Big (O). The Notation O(n) is the formal way to express the upper bound of an algorithms running time. It measures the worst case time complexity or the Longest amount of time an algorithm can possibly take to complete.



if

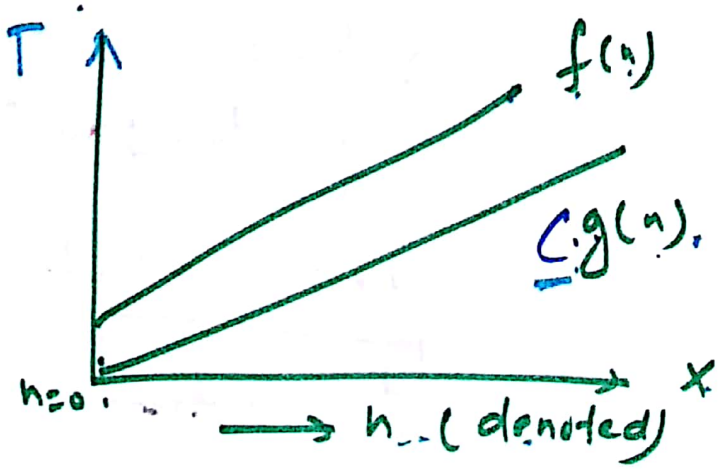
$f(n) \leq c.g(n)$

Where 'c' is constant

$$f(n) = O(g(n))$$

Ω (Omega Notation)

The notation $\Omega(n)$ is the formal way to express the Lower bound of an algorithms running time it measures the best Case Time Complexity.



If $f(n) \geq c \cdot g(n)$ where c is constant

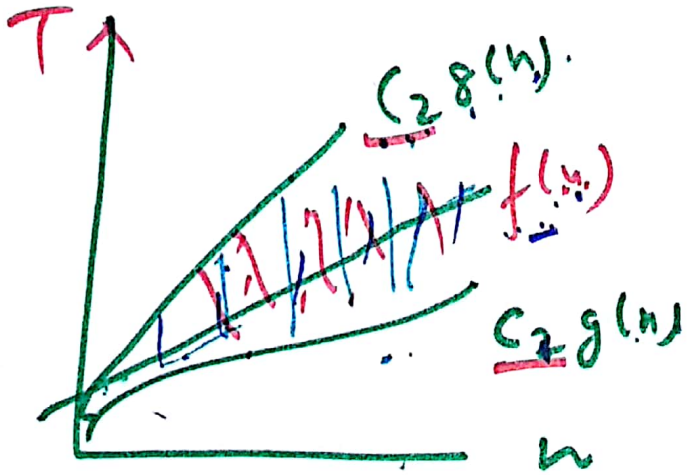
$$f(n) = \Omega(g(n))$$

Theta (Θ)

The notation $\Theta(n)$ is the formal way to express both the Lower bound and upper Bound of an algorithms running time. Also known as

Tight Bound.

if



if $c_1 g(n) \leq f(n) \leq c_2 g(n)$.

$$f(n) = \Theta(g(n))$$

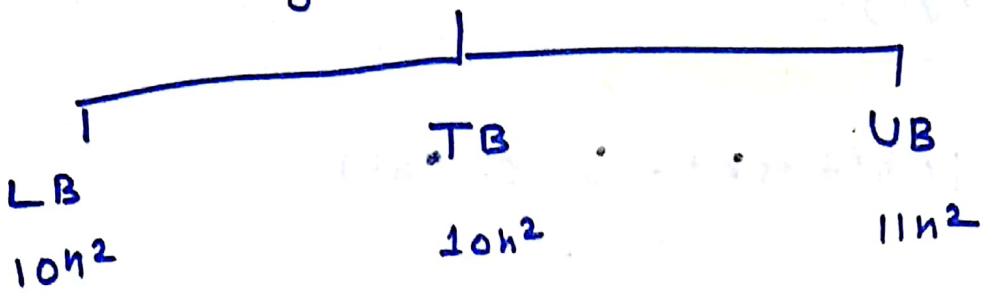
Q. find the Upper Bound, Lower Bound and tight Bound range for the following function (5)

- $10n^2 + 4n + 2$

Soln:- consider

$10n^2 + 4n + 2$

$g(n) = n^2$



(i) Big-oh notation:-

$$f(n) \leq c \cdot g(n)$$

$$10n^2 + 4n + 2 \leq c \cdot 10n^2$$

$$c = 11$$

~~$11n^2 + 4n + 2$~~

$$11n^2 + 4n + 2 \leq 11n^2$$

for $n=1$

$$11 \times 1^2 + 4 \times 1 + 2 \leq 11 \times 1^2$$

$$16 \leq 11 \text{ false}$$

for $n=2$

$$11 \times 2^2 + 4 \times 2 + 2 \leq 11 \times 2^2$$

$$50 \leq 44 \text{ false}$$

for $n=3$

$$11 \times 3^2 + 4 \times 3 + 2 \leq 11 \times 3^2$$

$$11 \times 9 + 12 + 2 \leq 11 \times 9$$

$$113 \leq 99 \text{ false}$$

for $n=4$

$$11 \times 4^2 + 4 \times 4 \leq 11 \times 4^2$$

$$178 \leq 176 \text{ false}$$

for $n=5$

$11\times5^2+4\times5+2 \leq 11\times5^2$

$272 \leq 275$ true

$\therefore f(n) = O(g(n))$

$10n^2+4n+2 = O(n^2) \ \forall\ n \geq 5,\ c=11$

Omega (Ω):-

$f(n) \geq c \cdot g(n)$

$10n^2+4n+2 \geq c \cdot (n^2)$

$c=10$

$10n^2+4n+2 \geq 10 \cdot n^2$

for $n=1$

$16 \geq 10$ - True.

$10n^2+4n+2 = \Omega(n^2) \ \forall\ n \geq 1, c=10$

Theta (Θ):- $c_1 g(n) \leq f(n) \leq c_2 \cdot g(n)$

$c_1 n^2 \leq 10n^2+4n+2 \leq c_2 n^2$

$10n^2 \leq 10n^2+4n+2 \leq 11n^2$

for $n=1$ $10 \leq 16 \leq 11$ false

for $n=2$ $40 \leq 50 \leq 44$ false

for $n=3$ $90 \leq 104 \leq 99$ false

for $n=4$ $160 \leq 178 \leq 176$ false

for $n=5$ $250 \leq 272 \leq 275$ true

$f(n) = O(g(n))$

$10n^2+4n+2 = \Theta(n^2) \ \forall\ n \geq 5,\ c_1 = 10,\ c_2 = 11$